

An Application of Convex Optimization to Nonuniform Defocus Removal

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Abstract.

We study the single-frame anisoplanatic deconvolution problem with the data image degraded by nonuniform defocus effects, called the *Nonuniform Defocus* (ND) problem. The problem naturally arises in many imaging applications such as reading traffic signs for self-driving vehicles, yet little has been known about its mathematical solutions. In this paper, we investigate an effective solution for the ND problem in the framework of convex optimization and analyze its correctness, potential from the theoretical, numerical and experimental perspectives. To achieve this goal, a number of research tasks in both computational methods and imaging science are conducted. Starting from being motivated by practical applications, we study the nonuniform defocus effects and construct a practically relevant imaging model for such images. Going from the physical model to the mathematical model, we formulate the inverse imaging problem and then the corresponding optimization problem. Progressing with solution methods, we provide the mathematical components for implementing almost every state-of-the-art convex optimization method, in particular, we adopt the algorithmic scheme introduced by Beck and Teboulle known as FISTA to the ND problem, called the *NDR algorithm*. Continuing towards theoretical and numerical analysis, we carefully study the physical and mathematical properties of the ND problem to prove global convergence of the NDR algorithm without any unverifiable technical assumptions and numerically analyze its important characteristics to show its superior performance compared to other relevant computational methods. Closing the loop of our research, we demonstrate the physical relevance of the ND problem model and the potential of the NDR algorithm on experimental images. The ND problem considers the more challenging case of single-frame anisoplanatic deconvolution where the subimages of object parts in different defocus zones cannot be separated from the overall image, for example, due to their overlapping boundaries. This underlying condition is a main difference of this survey compared to the other works on anisoplanatic deconvolution where the data image captures several objects with different, yet approximately constant defocus values. Therefore, solution methods partitioning the image into isoplanatic zones are not applicable to the ND problem and those such as the NDR algorithm handling the entire data image need to be developed and analyzed. Improving the performance of the NDR algorithm on experimental images and implementing it into real-world applications remain significant challenges and will be questions for our further research.